

# Antikaon production in nucleon-nucleon reactions near threshold\*

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## Abstract

The antikaon production cross section from nucleon-nucleon reactions near threshold is studied in a meson exchange model. We include both pion and kaon exchange, but neglect the interference between the amplitudes. In case of pion exchange the antikaon production cross section can be expressed in terms of the antikaon production cross section from a pion-nucleon interaction, which we take from the experimental data if available. Otherwise, a  $K^*$ -resonance exchange model is introduced to relate the different reaction cross sections. In case of kaon exchange the antikaon production cross section is related to the elastic  $KN$  and  $\bar{K}N$  cross sections, which are again taken from experimental measurements. We find that the one-meson exchange model gives a satisfactory fit to the available data for the  $NN \rightarrow NNK\bar{K}$  cross section at high energies. We compare our predictions for the cross section near threshold with an earlier empirical parameterization and that from phase space models.

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\*Supported by BMBF, GSI Darmstadt and Forschungszentrum Jülich GmbH

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# 1 Introduction

All data on antikaon production cross sections in nucleon-nucleon interactions are taken at high energies [1]. However, when studying antikaon production from heavy-ion collisions at energies per nucleon below the nucleon-nucleon threshold, cross sections slightly above the elementary threshold  $\sqrt{s_0} = 2m_N + 2m_K$  are needed [2].

The particular interest in antikaon production in nucleus-nucleus collisions at subthreshold energies arises from the expectation that it offers the possibility to study the in-medium properties of an antikaon at high baryon density. Indeed, in Ref. [2] it has been shown within a relativistic transport model [3, 4], that the antikaon yield can increase by an order of magnitude if its mass is allowed to decrease in dense matter. The experimental data on subthreshold  $K^-$ -production from heavy-ion collisions at GSI [5, 6] seem to support such a scenario.

In fact, according to studies based on effective chiral Lagrangians the antikaon mass should be reduced significantly in the medium due to the attractive scalar and vector potentials [7, 8, 9, 10]. A strong attractive antikaon potential is also consistent with the kaonic atom data [11]. The extrapolation of this attractive potential to high baryon density has led to the suggestion of a possible kaon condensate in neutron stars [12], which then might even lead to the formation of mini black holes in galaxies [13].

It is thus of great interest to learn from heavy-ion collisions at subthreshold energies about the antikaon properties at high densities. Although the relativistic transport model has been quite successful in explaining the experimental data on subthreshold  $K^+$  [14, 15, 16] and antiproton [16, 17, 18] production, the results obtained in Ref. [2] for subthreshold antikaon production depend on the input elementary antikaon production cross section from a nucleon-nucleon interaction as well as on the treatment of antikaon absorption by nucleons. In Ref. [2] the elementary antikaon production cross section in the parameterization introduced by Zwermann and Schürmann [19] (based on data from high energy proton-proton scattering) has been used. The accuracy of this parameterization near threshold is not known. Preliminary data on  $K^-$ -production from proton-proton reactions near threshold at COSY [20] seem to indicate a much smaller cross section than the one suggested by Ref. [19]. In order to extract more reliable information on antikaon in-medium properties from heavy-ion collisions one thus requires a good knowledge on the antikaon production cross section from the nucleon-nucleon interaction near threshold.

In the present paper we propose a meson exchange model to evaluate the antikaon production cross section from a nucleon-nucleon interaction. A similar approach (based on Ref. [21]) has also been used to evaluate the kaon production cross section from a nucleon-nucleon interaction in Ref. [22]. It

has been found that this model gives a more reliable cross section near threshold than the parameterization based on  $K^+$ -data from high energies [23, 24].

## 2 Pion exchange

The Feynman diagram for antikaon production from a nucleon-nucleon interaction  $NN \rightarrow NNK\bar{K}$  in the one-pion exchange model is shown in Fig. 1a). Following the derivation in Refs. [21, 22], the isospin-averaged cross section for this process can be written as

$$\bar{\sigma}(NN \rightarrow NNK\bar{K}; \sqrt{s}) = C \frac{m_N^2}{4\pi^2 p_i^2 s} \int_{W_{min}}^{W_{max}} dW W^2 k \frac{f_{\pi NN}^2}{m_\pi^2} \int_{t_-}^{t_+} dt F_\pi^4(t) \frac{1}{(t - m_\pi^2)^2} \bar{\sigma}(\pi N \rightarrow NK\bar{K}; W, t), \quad (1)$$

where  $\sqrt{s}$  is the center-of-mass energy of the colliding nucleons. The energy of the pion-nucleon system in their center-of-mass is given by  $W$ , with values between

$$W_{min} = 2m_K + m_N, \quad \text{and} \quad W_{max} = \sqrt{s} - m_N, \quad (2)$$

where  $m_N$  and  $m_K$  denote the masses of the nucleon and kaon, respectively. The squared four-momentum transfer from the initial to the final nucleon is denoted by  $t$  and varies between the two limits,

$$t_\pm = 2m_N^2 - 2E_i E_f \pm 2p_i p_f, \quad (3)$$

where  $E_i$ ,  $p_i$  are the energy and momentum of the initial nucleons in the center-of-mass frame, while  $E_f$ ,  $p_f$  are those for the final nucleons. They are related to  $\sqrt{s}$  by

$$p_i^2 = \lambda(s, m_N^2, m_N^2) \quad \text{and} \quad p_f^2 = \lambda(s, W^2, m_N^2), \quad (4)$$

with

$$\lambda(x, y, z) = \frac{(x - y - z)^2 - 4yz}{4x}. \quad (5)$$

Similarly, the three-momentum  $k$  of the exchanged pion is related to  $W$  and its mass  $m_\pi$  by  $k^2 = \lambda(W^2, m_N^2, m_\pi^2)$ . The pseudovector pion-nucleon coupling is denoted by  $f_{\pi NN}$  and has a value of  $\approx 1$ . To take into account the off-shell nature of the exchanged pion, we introduce (as in Ref. [25]) at the  $\pi NK\bar{K}N$  vertex a pion form factor similar to that in the  $\pi NN$  vertex,

$$F_\pi(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}, \quad (6)$$

with  $\Lambda$  denoting the cutoff parameter.

### 3 $\pi N \rightarrow NK\bar{K}$

In Eq. (1),  $\bar{\sigma}(\pi N \rightarrow NK\bar{K}; W, t)$  is the isospin-averaged on-shell  $\bar{K}$  production cross section from a  $\pi N$  interaction. The coefficient  $C$  in Eq. (1) depends on how different isospin channels for this reaction are related. One possible model for this reaction is shown in Fig. 1c), in which the pion annihilates with a virtual pion from the nucleon to produce the  $K\bar{K}$  pair via the exchange of a  $K^*$ -resonance. For on-shell pions, this process has been shown to give a reasonable description of  $K\bar{K}$  production from pion-pion interactions [26]. For simplicity, we neglect in the present work the off-shell effects by dropping the  $t$ -dependence in  $\bar{\sigma}(\pi N \rightarrow NK\bar{K}; W, t)$  and using thus the empirical on-shell cross section.

Experimental data on antikaon production from pion-nucleon interactions are available for the following reactions [1]:  $\pi^- p \rightarrow pK^0 K^-$ ,  $\pi^- p \rightarrow nK^+ K^-$ ,  $\pi^- p \rightarrow nK^0 \bar{K}^0$ ,  $\pi^+ p \rightarrow pK^+ \bar{K}^0$ ,  $\pi^+ n \rightarrow pK^+ K^-$ ,  $\pi^+ n \rightarrow pK^0 \bar{K}^0$ , and  $\pi^+ n \rightarrow nK^+ \bar{K}^0$  (cf. Figs. 2 and 3). We first parameterize the experimental  $\pi^- p \rightarrow pK^0 K^-$  cross section by the expression,

$$\sigma(\pi^- p \rightarrow pK^0 K^-) = 1.121 \left(1 - \frac{s_0}{s}\right)^{1.86} \left(\frac{s_0}{s}\right)^2 [mb], \quad (7)$$

where  $\sqrt{s}$  is the invariant mass of the  $\pi N$  system and  $\sqrt{s_0} = m_N + 2m_K$ . The results of the fit are shown in Fig. 2a) by the solid curve in comparison to the data from Ref. [1]. Instead of fitting the other cross sections separately, we explore the isospin symmetry of the Feynman diagram shown in Fig. 1c), leading to the following relations among the cross sections:

$$\begin{aligned} 2\sigma(\pi^+ p \rightarrow pK^+ \bar{K}^0) &= 2\sigma(\pi^+ n \rightarrow nK^+ \bar{K}^0) = \sigma(\pi^+ n \rightarrow pK^+ K^-) = \\ \sigma(\pi^+ n \rightarrow pK^0 \bar{K}^0) &= \sigma(\pi^0 p \rightarrow nK^+ \bar{K}^0) = 4\sigma(\pi^0 p \rightarrow pK^+ K^-) = \\ 4\sigma(\pi^0 p \rightarrow pK^0 \bar{K}^0) &= \sigma(\pi^0 n \rightarrow pK^0 K^-) = 4\sigma(\pi^0 n \rightarrow nK^+ K^-) = \\ 4\sigma(\pi^0 n \rightarrow nK^0 \bar{K}^0) &= 2\sigma(\pi^- p \rightarrow pK^0 K^-) = \sigma(\pi^- p \rightarrow nK^+ K^-) = \\ \sigma(\pi^- p \rightarrow nK^0 \bar{K}^0) &= 2\sigma(\pi^- n \rightarrow nK^0 K^-). \end{aligned} \quad (8)$$

All cross sections are thus related to  $\sigma(\pi^- p \rightarrow pK^0 K^-)$ ; they are shown in Figs. 2b), 2c), and 3) by the solid curves, and it is seen that they agree quite well with the data. We, therefore, believe that the isospin relations are properly treated via the diagram in Fig. 1c) and that  $K^*$ -resonance exchange is the dominant mechanism. For comparison, we also show in Fig. 2b) (dashed curve) the parameterization introduced in Ref. [27], i.e.,

$$\sigma(\pi^- p \rightarrow nK^0 \bar{K}^0) = \frac{0.158(\sqrt{s} - \sqrt{s_0})^2}{0.1735 + (\sqrt{s} - \sqrt{s_0})^3} [mb]. \quad (9)$$

We find that our parameterization fits the data better near threshold than that of Ref. [27].

From Eq.(8) we can also derive the isospin-averaged cross section for  $\pi N \rightarrow NK\bar{K}$ , which then is given by

$$\bar{\sigma}(\pi N \rightarrow NK\bar{K}) = 3\sigma(\pi^- p \rightarrow pK^0 K^-). \quad (10)$$

This cross section is useful for transport models that do not explicitly include the isospin degree of freedom.

With the model shown in Fig. 1c) for the reaction  $\pi N \rightarrow NK\bar{K}$ , the coefficient  $C$  in Eq. (1) has a value of 7/8. Now substituting the isospin-averaged  $\pi N \rightarrow NK\bar{K}$  cross section in Eq. (1), we can evaluate the isospin-averaged cross section for antikaon production from nucleon-nucleon interactions. Experimental data are available for the reactions  $pp \rightarrow pnK^+ \bar{K}^0$  and  $pp \rightarrow ppK^0 \bar{K}^0$  [1] at high energies. From isospin symmetry, they can be related to the isospin-averaged cross section by

$$\sigma(pp \rightarrow pnK^+ \bar{K}^0) = 4\sigma(pp \rightarrow ppK^0 \bar{K}^0) = \frac{16}{21}\bar{\sigma}(NN \rightarrow NNK\bar{K}). \quad (11)$$

We use a monopole cutoff  $\Lambda=1.2$  GeV in Eq.(6), which is taken from the analysis of  $K^+$ -meson production [22]. Our results calculated with the one-pion exchange model according to Eq.(1) are shown in Fig. 4 by the dashed lines and are seen to underestimate the experimental data by about a factor 3-4.

## 4 Kaon exchange

Kaon exchange also contributes to antikaon production from a nucleon-nucleon interaction as shown in Fig.1b. Neglecting interferences, the cross section can be expressed in terms of the product of elastic  $KN$  and  $\bar{K}N$  cross sections [21, 28]. Both isospin ( $I=1$  and  $I=0$ ) amplitudes contribute to the latter reactions. If their interference is neglected, we can relate the  $I=1$  and  $I=0$  cross sections to the empirical ones [29], i.e.

$$\begin{aligned} \sigma_1(KN \rightarrow KN) &= \sigma(K^+ p \rightarrow K^+ p) \\ \sigma_0(KN \rightarrow KN) &= \sigma(K^+ n \rightarrow K^+ n) + \sigma(K^+ n \rightarrow K^0 p) \\ &\quad - \sigma(K^+ p \rightarrow K^+ p) \end{aligned} \quad (12)$$

and

$$\begin{aligned} \sigma_1(\bar{K}N \rightarrow \bar{K}N) &= \sigma(K^- n \rightarrow K^- n) \\ \sigma_0(\bar{K}N \rightarrow \bar{K}N) &= \sigma(K^- p \rightarrow K^- p) + \sigma(K^- p \rightarrow K^0 n) \\ &\quad - \sigma(K^- n \rightarrow K^- n). \end{aligned} \quad (13)$$

The cross sections on the right hand side of Eqs. (12) and (13) are available empirically and used in the parameterizations of Ref. [29]. We note that

the results for  $\sigma_0(KN)$  and  $\sigma_1(KN)$  are in reasonable agreement with the calculations on  $KN$  elastic cross sections performed recently by Hoffmann et al. [30].

In terms of the above cross sections we have

$$\begin{aligned} \sigma(pp \rightarrow pp K^0 \bar{K}^0; \sqrt{s}) &= \frac{1}{2\pi^3 p_i^2 s} \int_{W_{min}}^{W_{max}} dW W^2 k_1 [\sigma_1(KN \rightarrow KN; W) \\ &+ \sigma_0(KN \rightarrow KN; W)] \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_1(\bar{K}N \rightarrow \bar{K}N; U) \\ &\times \int_{t_-}^{t_+} dt F_K^4(t) \frac{1}{(t - m_K^2)^2}. \end{aligned} \quad (14)$$

In Eq. (14)  $\sqrt{s}$  is again the center-of-mass energy of the colliding nucleons,  $W$  is the energy of the kaon-nucleon system in their center-of-mass, and  $U$  is that of the antikaon-nucleon system. The allowed values for  $W$  and  $U$  are given by

$$\begin{aligned} W_{min} &= m_K + m_N, & W_{max} &= \sqrt{s} - m_N - m_K, \\ U_{min} &= m_K + m_N, & \text{and } U_{max} &= \sqrt{s} - W. \end{aligned} \quad (15)$$

The three-momenta  $k_1$  and  $k_2$  of the exchange kaon in the  $KN$  and  $\bar{K}N$  system are related to  $W$  and  $U$  by

$$k_1^2 = \lambda(W^2, m_N^2, m_K^2) \quad \text{and} \quad k_2^2 = \lambda(U^2, m_N^2, m_K^2). \quad (16)$$

The squared four-momentum transfer from the initial nucleon to the final  $KN$  system is denoted by  $t$  and varies between the two limits,

$$t_{\pm} = W^2 + m_N^2 - 2E_i E_f \pm 2p_i p_f, \quad (17)$$

where  $E_i$ ,  $p_i$  are the energy and momentum of the initial nucleons in the center-of-mass frame defined as before, while  $E_f$ ,  $p_f$  are those for the final  $KN$  system and are given by

$$E_f = \frac{(s + W^2 - U^2)}{2\sqrt{s}} \quad \text{and} \quad p_f^2 = \lambda(s, W^2, U^2). \quad (18)$$

We account for the off-shellness of the exchanged kaon at both  $KN$  and  $\bar{K}N$  vertices by introducing a monopole form factor with the cutoff parameter  $\Lambda=1.0$  GeV, which is close to that used in Ref. [22].

Similarly, one can show that

$$\begin{aligned} \sigma(pp \rightarrow pn K^+ \bar{K}^0; \sqrt{s}) &= \frac{1}{2\pi^3 p_i^2 s} \\ &\left[ \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_1(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_1(\bar{K}N \rightarrow \bar{K}N; U) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_1(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_0(\bar{K}N \rightarrow \bar{K}N; U) \\
& + \frac{1}{2} \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_0(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_1(\bar{K}N \rightarrow \bar{K}N; U) \Big] \\
& \times \int_{t_-}^{t_+} dt F_K^4(t) \frac{1}{(t - m_K^2)^2}. \tag{19}
\end{aligned}$$

The isospin-averaged antikaon production cross section due to kaon exchange is then given by

$$\begin{aligned}
\bar{\sigma}(NN \rightarrow NNK\bar{K}; \sqrt{s}) &= \frac{1}{\pi^3 p_i^2 s} \\
& \left[ \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_1(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_1(\bar{K}N \rightarrow \bar{K}N; U) \right. \\
& + \frac{1}{2} \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_1(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_0(\bar{K}N \rightarrow \bar{K}N; U) \\
& + \frac{1}{2} \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_0(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_1(\bar{K}N \rightarrow \bar{K}N; U) \\
& \left. + \frac{1}{8} \int_{W_{min}}^{W_{max}} dW W^2 k_1 \sigma_0(KN \rightarrow KN; W) \int_{U_{min}}^{U_{max}} dU U^2 k_2 \sigma_0(\bar{K}N \rightarrow \bar{K}N; U) \right] \\
& \times \int_{t_-}^{t_+} dt F_K^4(t) \frac{1}{(t - m_K^2)^2}. \tag{20}
\end{aligned}$$

The contribution to antikaon production from the kaon exchange is found to be more important than that from the pion exchange. The total contributions from both (pion and kaon) exchanges are shown in Fig. 4 by the solid lines and are seen to reproduce reasonably well the experimental data.

Furthermore, our numerical results for the isospin averaged antikaon production cross section from the nucleon-nucleon interaction calculated with the pion and kaon exchange model can be parameterized by

$$\bar{\sigma}(NN \rightarrow NNK\bar{K}, \sqrt{s}) = 0.3 \left(1 - \frac{s_0}{s}\right)^{3.0} \left(\frac{s_0}{s}\right)^{0.8} [mb], \tag{21}$$

where  $\sqrt{s_0} = 2m_N + 2m_K$ . Again, this will be useful for transport model calculations without explicit treatment of the isospin degrees of freedom.

## 5 Discussion

We show in Fig. 5 the available parameterizations for the inclusive  $K^-$  production from  $pp$  collisions together with the experimental data [1]. We note, that our result (solid line) includes the contributions from pion and kaon exchanges, but accounts only for the exclusive reaction  $pp \rightarrow ppK^+K^-$ .

Obviously we underestimate the data at higher energies since here processes with one ( $\sqrt{s} - \sqrt{s_0} \gg m_\pi$ ) and more pions in the final state also contribute to the inclusive cross section.

For comparison, the dash-dotted line in Fig. 5 shows the results from Ref. [31] calculated with the statistical quark ROC model [32], while the dotted line is the parameterization from Ref. [33] based on phase space considerations. The dashed line in Fig. 5 is the parameterization from [19] and differs drastically from our results at low energies since it tries to fit the data point at about 0.1 GeV above threshold from Ref. [34], which does not follow the systematics implied by phase space considerations. It is thus very important to have experimental data at low energies to determine if our model is appropriate. In this respect, experiments currently being carried out at COSY (at 6 MeV above threshold) [35] are extremely useful.

## 6 Summary

In summary, we have introduced a meson exchange model for antikaon production from nucleon-nucleon interactions. This model allows us to express the cross section in terms of the off-shell production amplitudes from the pion-nucleon and kaon-nucleon interactions. Approximating the off-shell amplitudes by the measured on-shell amplitudes and neglecting their interferences, the  $NN \rightarrow NNK\bar{K}$  cross section can be easily calculated at low energies, where the cross section is needed for studying antikaon production in proton-nucleus and nucleus-nucleus collisions at subthreshold energies.

Comparing with earlier parameterizations, our predictions give a smaller cross section near threshold, roughly in line with the phase space considerations from Ref. [33]. It is thus important to have experimental data from nucleon-nucleon interactions at low energies to verify our predictions.

As a byproduct of our study we have found that the available data for the antikaon production cross section from the different isospin channels in the reaction  $\pi N \rightarrow NK\bar{K}$  are consistent with a model in which the pion interacts with a virtual pion from the nucleon through the exchange of a  $K^*$ -resonance. A simple parameterization is introduced and is able to account for all available data for this reaction.

We note that the study of antikaon production from subthreshold heavy-ion collisions requires the knowledge of both nucleon-nucleon and pion-nucleon production cross sections. In Ref. [2], the parameterization of Zwermann and Schürmann [19], which is a few orders of magnitude larger than ours near threshold, has been used; yet the results have indicated that a strong attractive antikaon potential is needed in order to explain the measured cross section from Ref. [5]. With a smaller elementary cross section according to our prediction, the antikaon medium effects should be even more pronounced.



# Acknowledgement

The authors like to acknowledge valuable discussions with U. Mosel throughout this study. C.M.K. was also supported in part by the National Science Foundation under Grant No. PHY-9509266 and the Alexander von Humboldt Foundation.

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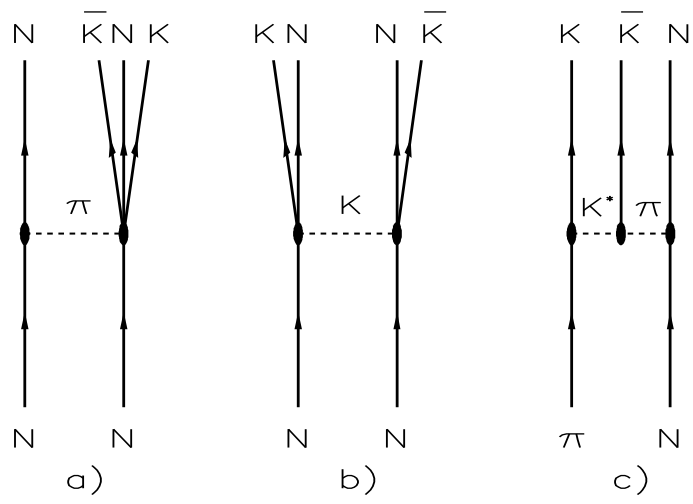


Figure 1: Feynman diagrams for the reaction  $NN \rightarrow NNK\bar{K}$  (a,b) and  $\pi N \rightarrow NK\bar{K}$  (c).

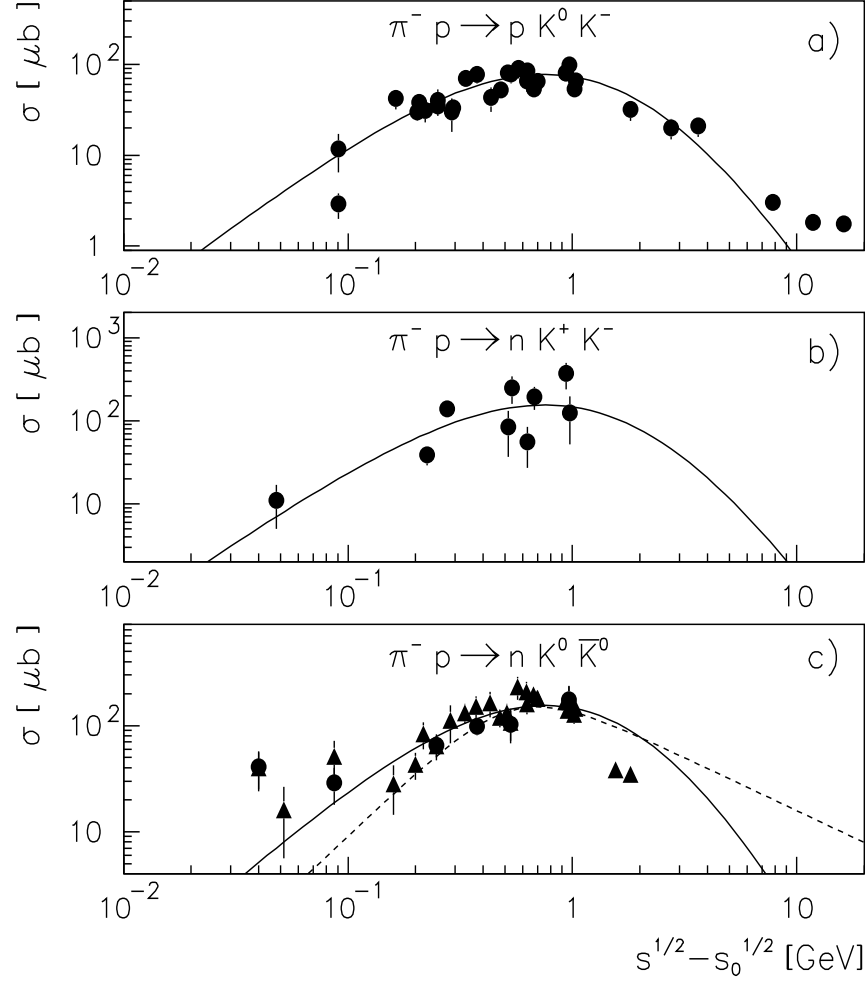


Figure 2: Cross sections for the reactions: (a)  $\pi^- p \rightarrow p K^0 K^-$ , (b)  $\pi^- p \rightarrow n K^+ K^-$ , and (c)  $\pi^- p \rightarrow n K^0 \bar{K}^0$ . The solid lines show our parameterization (Eqs. (7) and (8)), while the experimental data are from Ref. [1]. The dashed line is the parameterization (Eq. (9)) from Ref. [27].

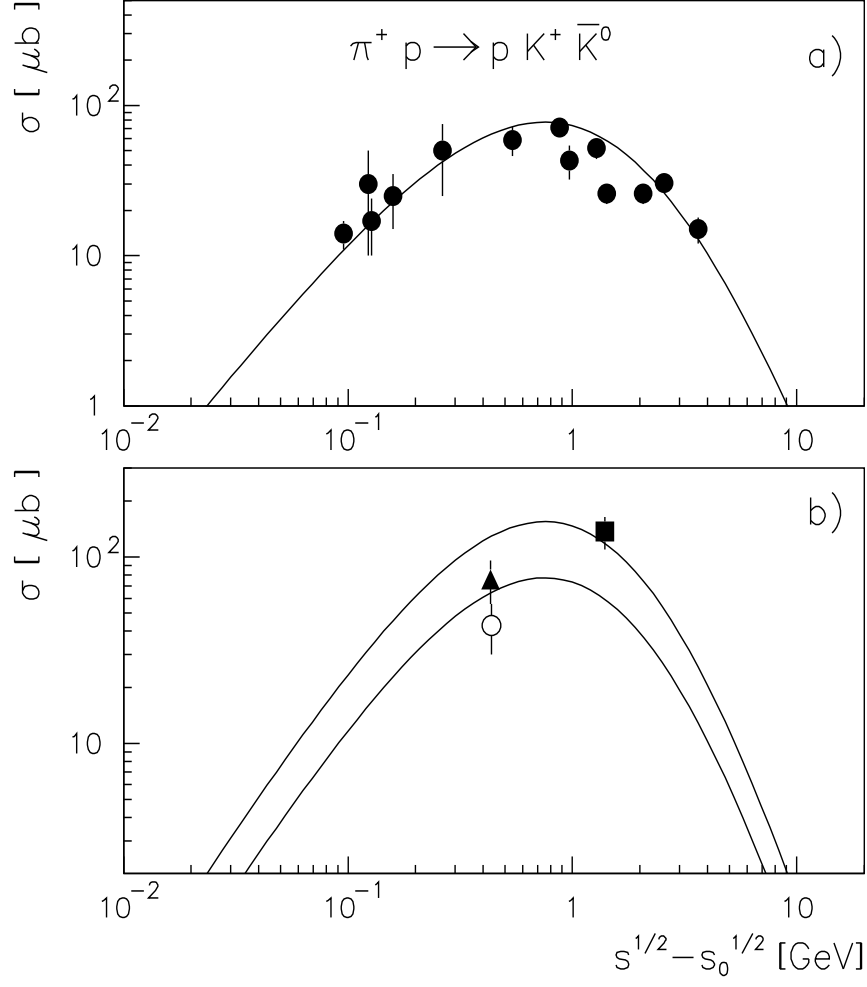


Figure 3: Same as Fig. 2 for the reactions: (a)  $\pi^+ p \rightarrow p K^+ \bar{K}^0$  and (b)  $\pi^+ n \rightarrow n K^+ K^-$  (square),  $\pi^+ p \rightarrow p K^0 \bar{K}^0$  (triangle), and  $\pi^+ n \rightarrow n K^+ \bar{K}^0$  (open circle). The lower solid line in (b) is our result for the reactions  $\pi^+ p \rightarrow p K^0 \bar{K}^0$  and  $\pi^+ n \rightarrow n K^+ \bar{K}^0$ , while the upper solid line in (b) is that for the reaction  $\pi^+ n \rightarrow n K^+ K^-$ .

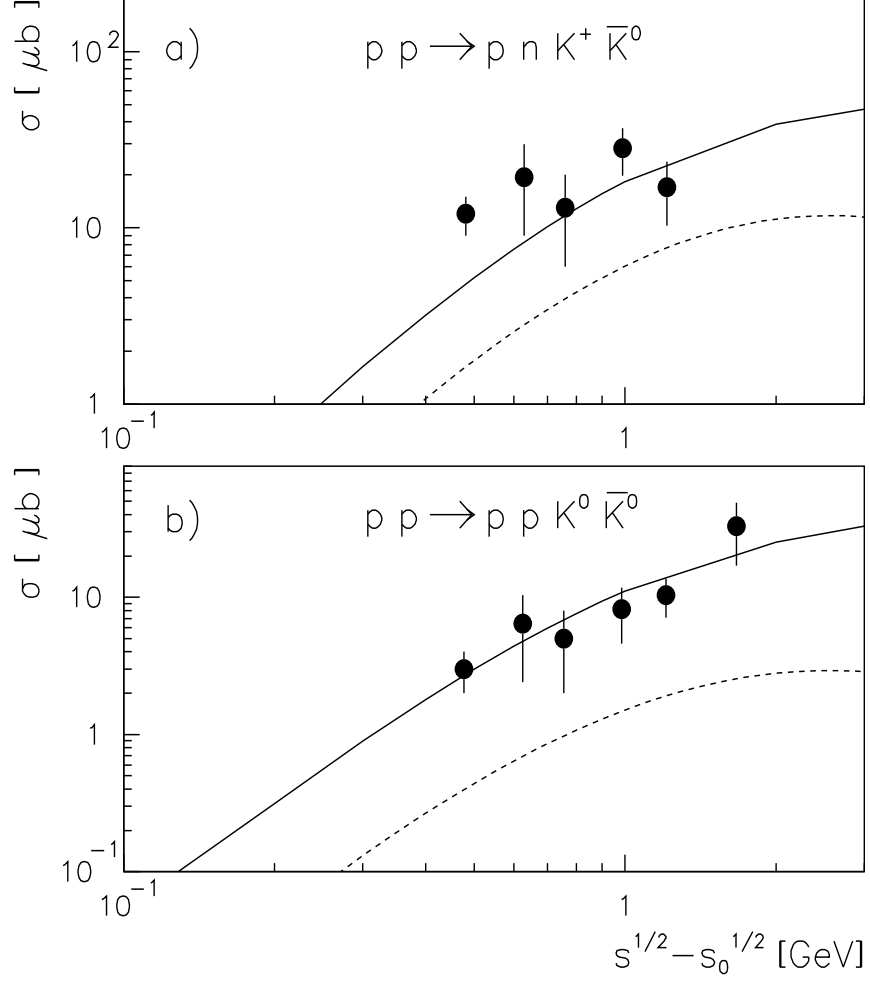


Figure 4: Cross sections for the reactions: (a)  $pp \rightarrow pnK^+\bar{K}^0$  and (b)  $pp \rightarrow ppK^0\bar{K}^0$ . The dashed lines show our results calculated with the one-pion exchange model according to Eqs. (1) and (11), while the solid lines are the total contribution from both pion and kaon exchange. The experimental data are from Ref. [1].

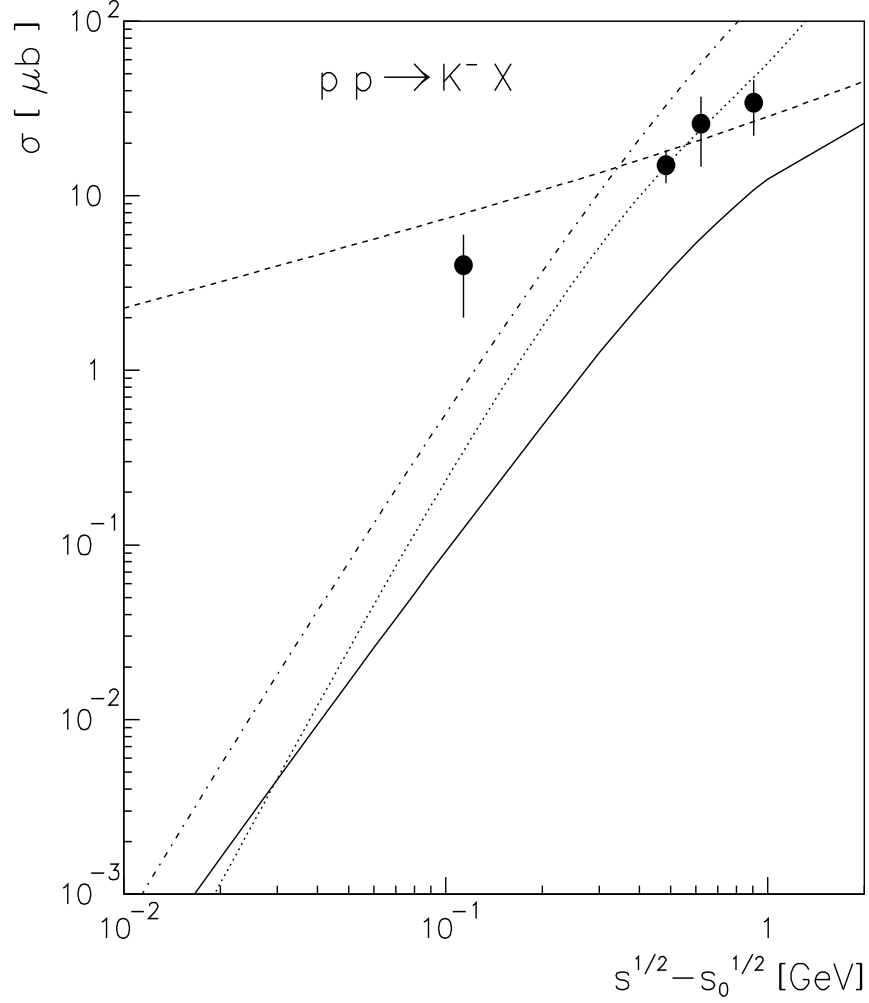


Figure 5:  $K^-$  inclusive production cross section from proton-proton interactions in comparison to the experimental data from Ref. [1]. The solid line shows our calculation for the exclusive reaction  $pp \rightarrow ppK^+K^-$ ; the dashed line represents the parameterization from Ref. [19], the dotted line is the parameterization from Ref. [33], and the dash-dotted line from the statistical ROC model of Refs. [31, 32].